2.3 — Cost Minimization

ECON 306 • Microeconomic Analysis • Fall 2022

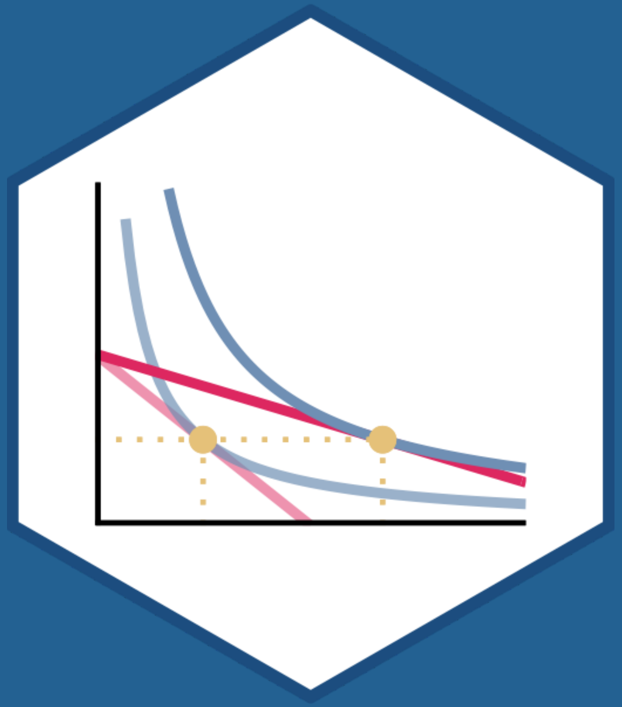
Ryan Safner

Associate Professor of Economics

safner@hood.edu

[ryansafner/microF22](https://github.com/ryansafner/microF22)

microF22.classes.ryansafner.com



Recall: The Firm's Two Problems

1st Stage: **firm's profit maximization problem**:

1. **Choose:** **< output >**

2. **In order to maximize:** **< profits >**

- We'll cover this later...first we'll explore:

2nd Stage: **firm's cost minimization problem**:

1. **Choose:** **< inputs >**

2. **In order to *minimize*:** **< cost >**

3. **Subject to:** **< producing the optimal output >**

- Minimizing costs $\iff$ maximizing profits

Solving the Cost Minimization Problem

The Firm's Cost Minimization Problem

- The **firm's cost minimization problem** is:

1. **Choose:** < inputs: l, k>

2. **In order to minimize:** < total cost: $wl + rk$ >

3. **Subject to:** < producing the optimal output: $q^* = f(l, k)$ >

The Cost Minimization Problem: Tools

- Our tools for firm's input choices:
- **Choice**: combination of inputs (l, k)
- **Production function/isoquants**: firm's technological constraints
  - How the *firm* trades off between inputs
- **Isocost line**: firm's total cost (for given output and input prices)
  - How the *market* trades off between inputs

The Cost Minimization Problem: Verbally

- The **firms's cost minimization problem**:

> choose a combination of l and k to minimize total cost that produces the optimal amount of output

The Cost Minimization Problem: Math

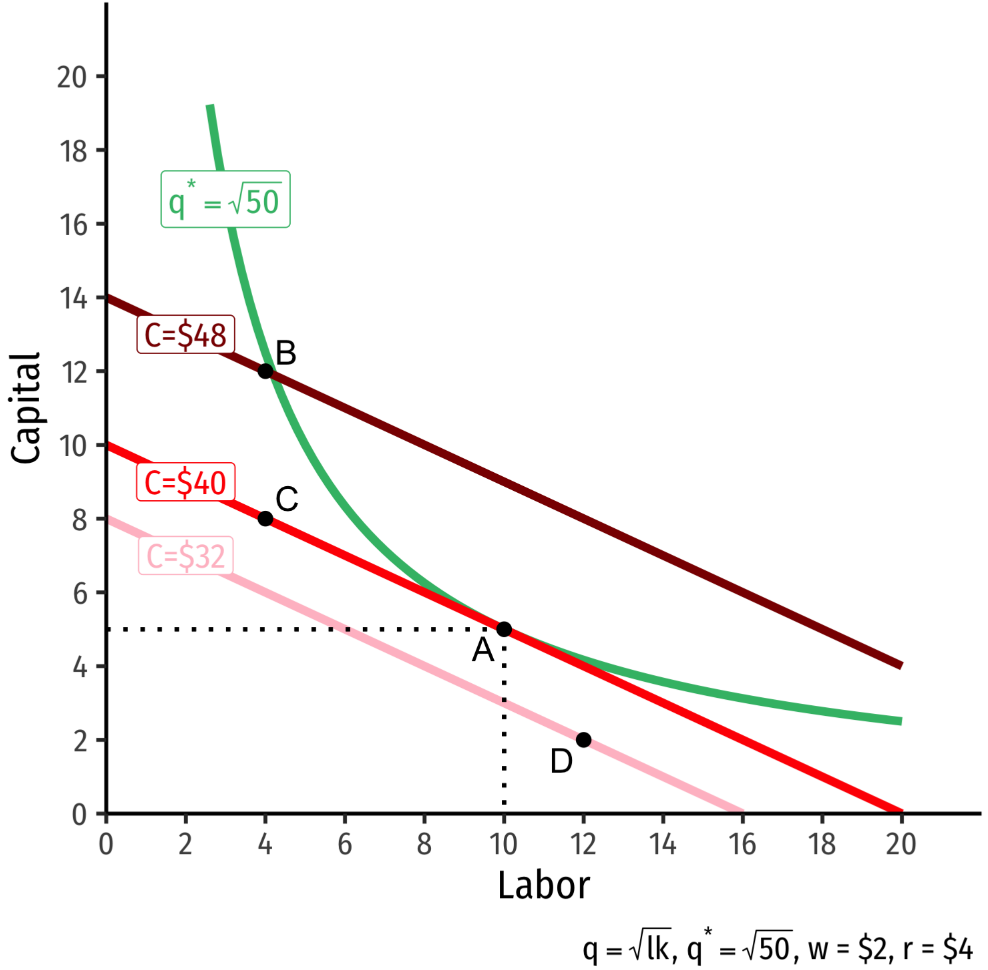
$$\min_{l,k} wl + rk$$

$$s.t. \quad q^* = f(l,k)$$

- This requires calculus to solve. We will look at **graphs** instead!

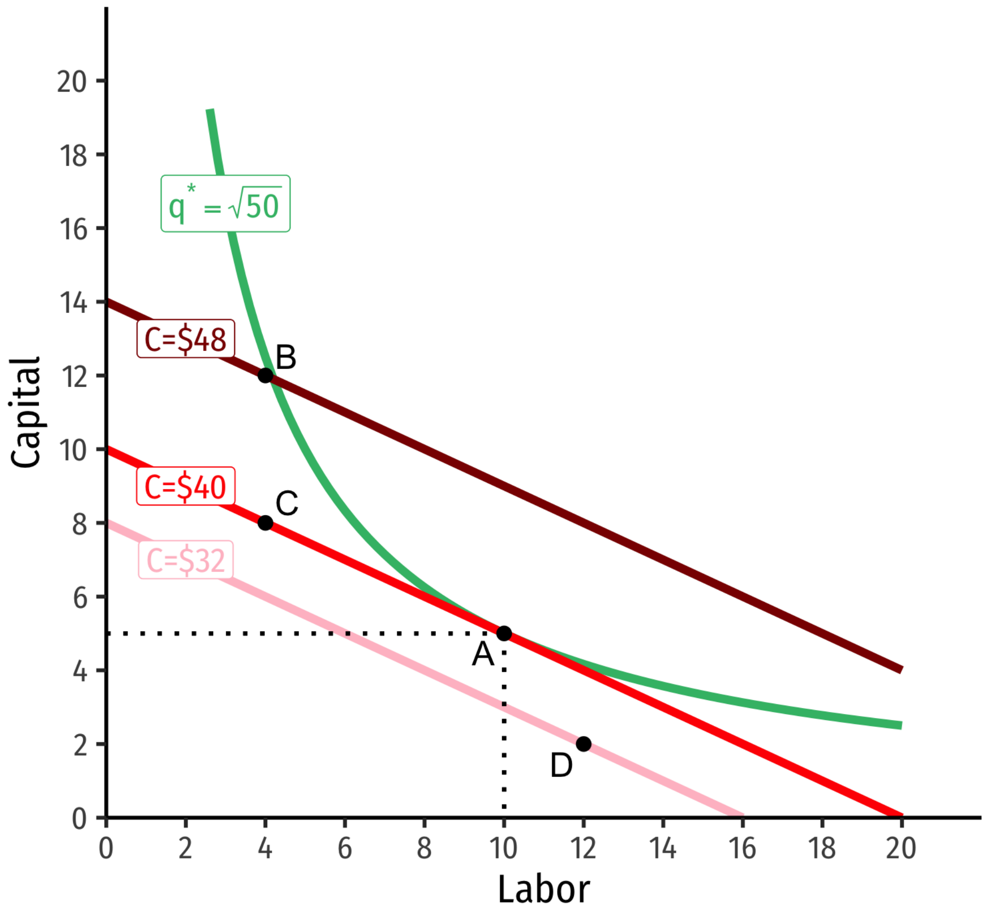
The Firm's Least-Cost Input Combination: Graphically

- Graphical solution: **Lowest** isocost line ***tangent* to desired** isoquant (A)



The Firm's Least-Cost Input Combination: Graphically

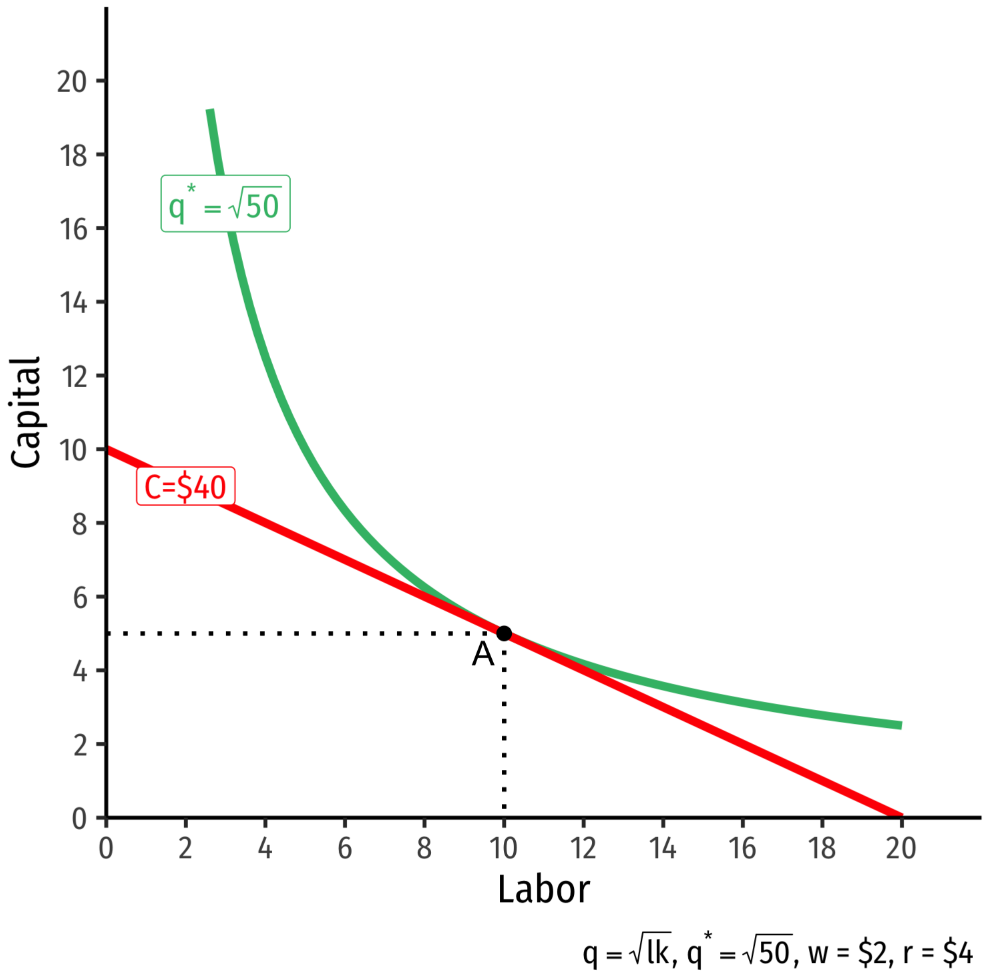
- Graphical solution: **Lowest** isocost line ***tangent* to desired** isoquant (A)
- B produces same output as A, but higher cost
- C is same cost as A, but does not produce desired output
- D is cheaper, does not produce desired output



$q = \sqrt{lk}$, $q^* = \sqrt{50}$, $w = \$2$, $r = \$4$

The Firm's Least-Cost Input Combination: Why A?

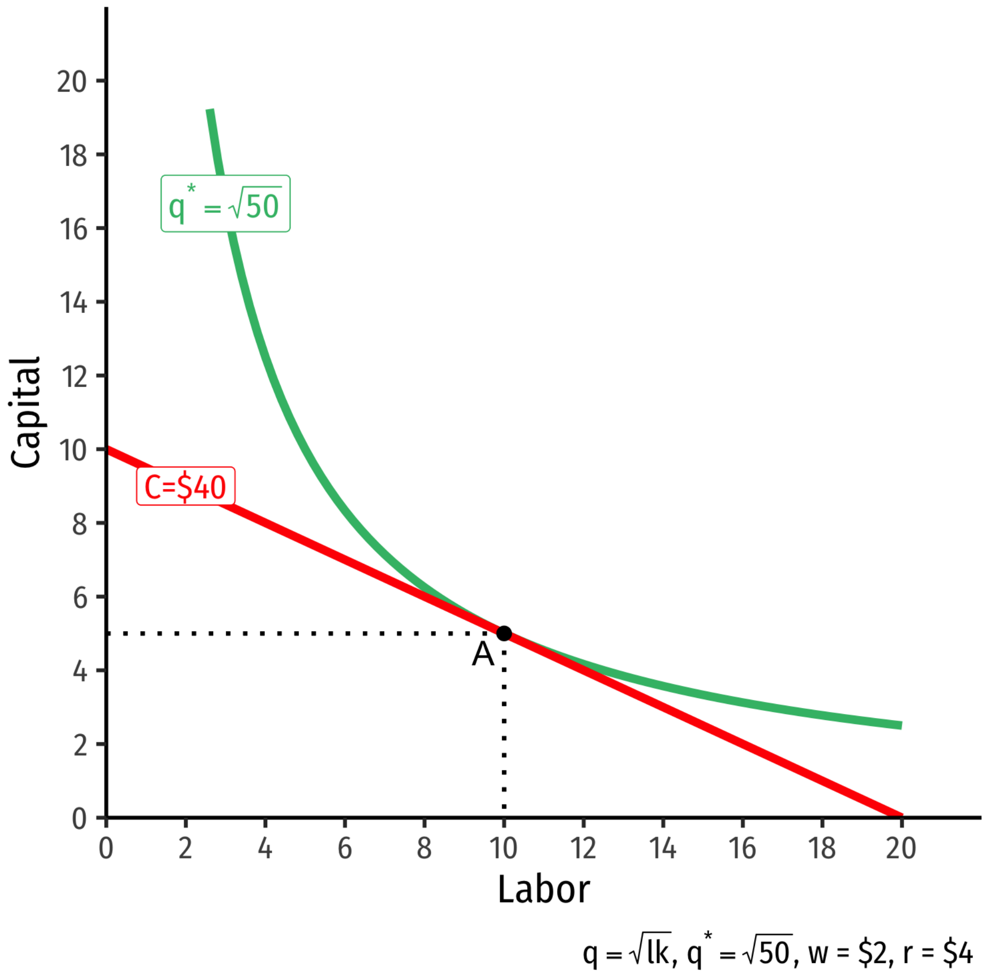
$$\color{green}{\text{Isoquant curve slope}} = \color{red}{\text{Isocost line slope}}$$



The Firm's Least-Cost Input Combination: Why A?

$$\text{Isoquant curve slope} = \text{Isocost line slope}$$
$$MRTS_{l,k} = \frac{w}{r}$$
$$\frac{MP_l}{MP_k} = \frac{w}{r}$$
$$0.5 = 0.5$$

- **Marginal benefit** = **Marginal cost**
  - **Firm** would exchange at same rate as **market**
- **No other combination of (l,k) exists at current prices & output that could produce $q^\star$ at lower cost!**

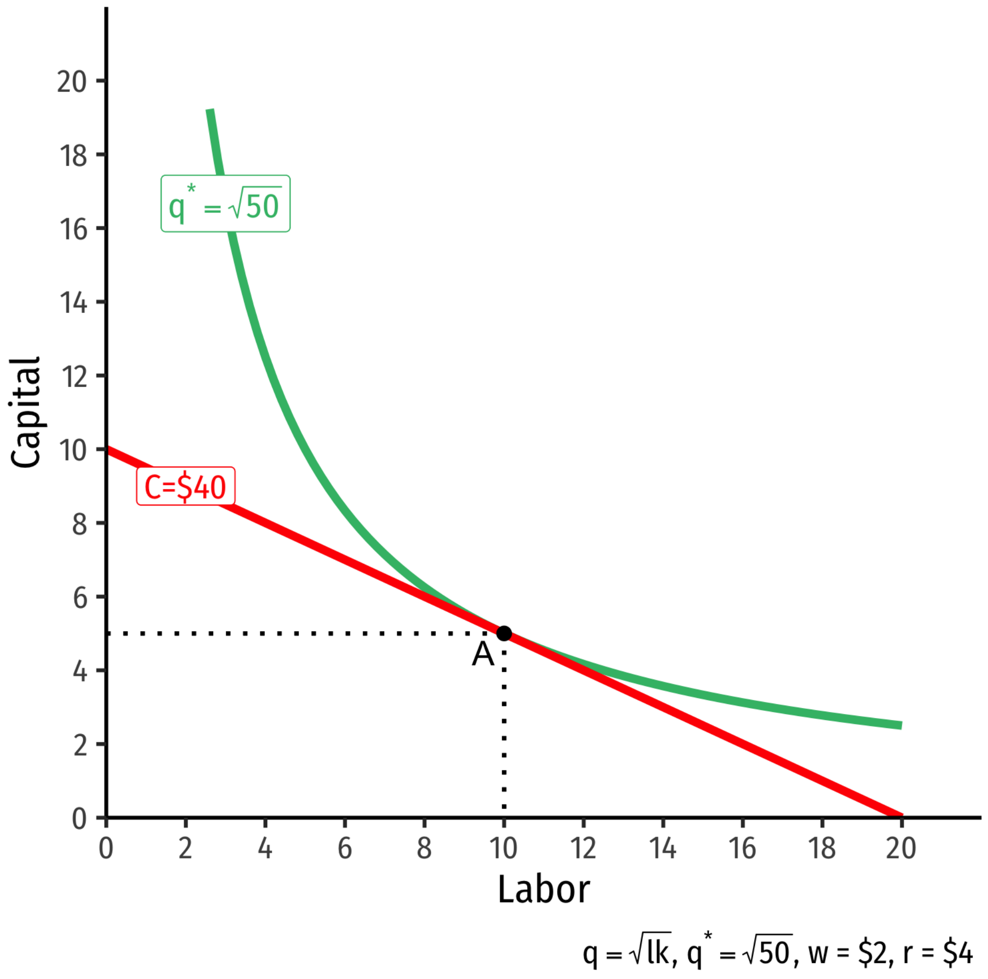


Two Equivalent Rules

Rule 1

$$\frac{MP_l}{MP_k} = \frac{w}{r}$$

- Easier for calculation (slopes)



Two Equivalent Rules

Rule 1

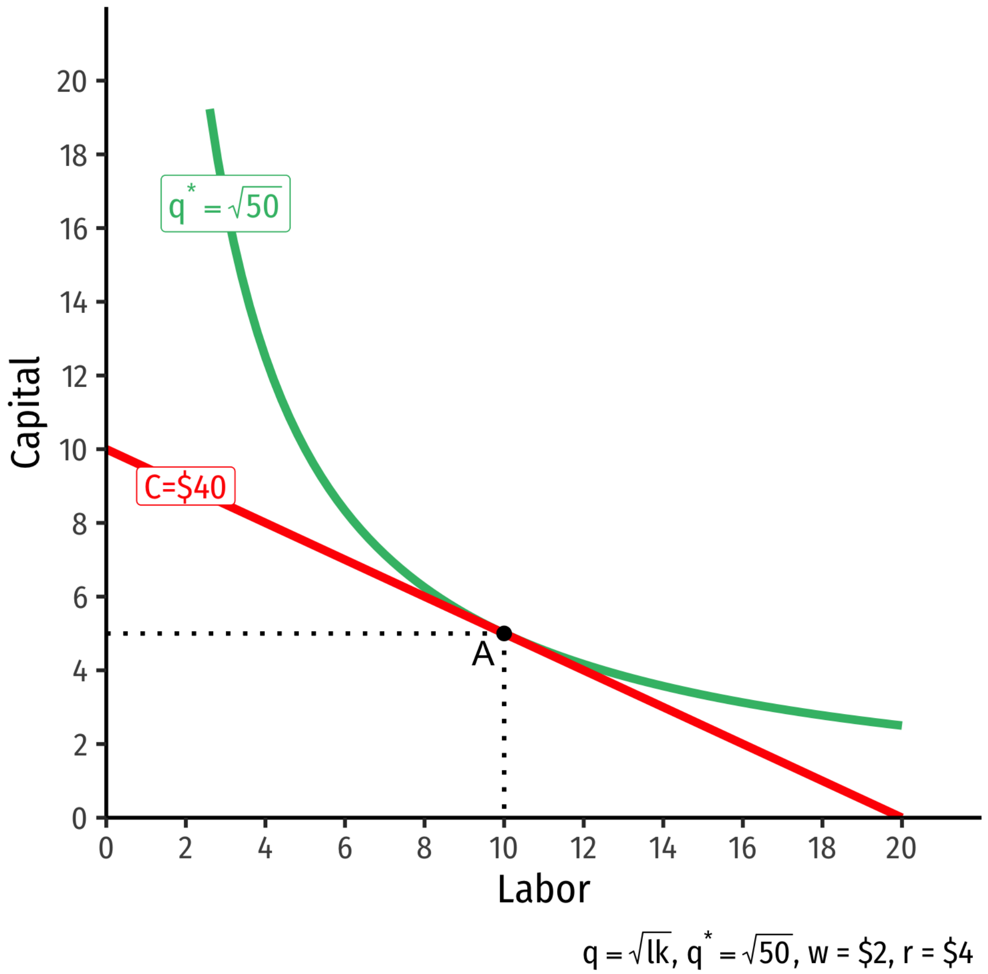
$$\frac{MP_l}{MP_k} = \frac{w}{r}$$

- Easier for calculation (slopes)

Rule 2

$$\frac{MP_l}{w} = \frac{MP_k}{r}$$

- Easier for intuition (next slide)



The Equimarginal Rule Again I

$$\frac{MP_l}{w} = \frac{MP_k}{r} = \cdots = \frac{MP_n}{p_n}$$

- **Equimarginal Rule**: the cost of production is minimized where the **marginal product per dollar spent** is **equalized** across all n possible inputs

- Firm will always choose an option that gives higher marginal product (e.g. if $MP_l > MP_k$)
  - But each option has a different cost, so we weight each option by its price, hence $\frac{MP_n}{p_n}$

The Equimarginal Rule Again II

- Any **optimum** in economics: no better alternatives exist under current constraints

- No possible change in your inputs to produce q^* that would lower cost

The Firm's Least-Cost Input Combination: Example

Example:

Your firm can use labor l and capital k to produce output according to the production function:

$$q = 2lk$$

The marginal products are:

$$MP_l = 2k$$
$$MP_k = 2l$$

You want to produce 100 units, the price of labor is \$10, and the price of capital is \$5.

1. What is the least-cost combination of labor and capital that produces 100 units of output?
2. How much does this combination cost?

Returns to Scale

Returns to Scale

- The **returns to scale** of production: change in output when **all** inputs are increased **at the same rate** (scale)

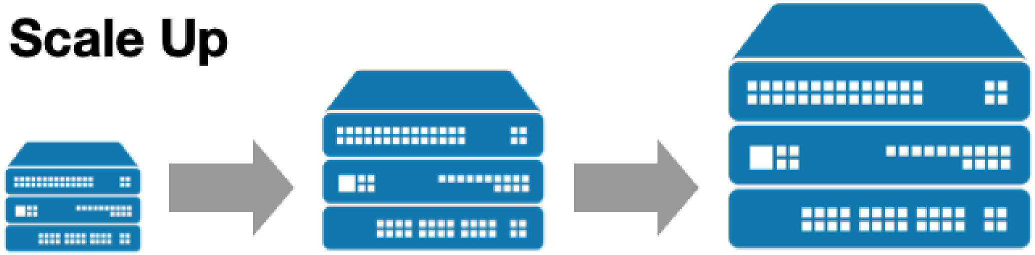
Returns to Scale

- The **returns to scale** of production: change in output when **all** inputs are increased **at the same rate** (scale)

- **Constant returns to scale**: output increases at **same proportionate rate** to inputs change
    - e.g. double all inputs, output doubles

Returns to Scale

- The **returns to scale** of production: change in output when **all** inputs are increased at the same rate (scale)
- **Constant returns to scale**: output increases at same proportionate rate to inputs change
    - e.g. double all inputs, output doubles
- **Increasing returns to scale**: output increases more than proportionately to inputs change
    - e.g. double all inputs, output *more than* doubles



Returns to Scale

- The **returns to scale** of production: change in output when **all** inputs are increased **at the same rate** (scale)

- **Constant returns to scale**: output increases at **same proportionate rate** to inputs change
  - e.g. double all inputs, output doubles

- **Increasing returns to scale**: output increases **more than proportionately** to inputs change[†]
  - e.g. double all inputs, output *more than* doubles

- **Decreasing returns to scale**: output increases **less than proportionately** to inputs change
  - e.g. double all inputs, output *less than* doubles

[†] See my new newsletter [Increasing Returns] for more on the importance of this idea

Returns to Scale: Example

Example: Do the following production functions exhibit *constant* returns to scale, *increasing* returns to scale, or *decreasing* returns to scale?

1. $q = 4l + 2k$
2. $q = 2lk$
3. $q = 2l^{0.3}k^{0.3}$

Returns to Scale: Cobb-Douglas

- One reason Cobb-Douglas functions are great: easy to determine returns to scale:

$$q = Ak^{\alpha}l^{\beta}$$

- $\alpha + \beta = 1$: constant returns to scale
- $\alpha + \beta > 1$: increasing returns to scale
- $\alpha + \beta < 1$: decreasing returns to scale
- Note this trick *only* works for Cobb-Douglas functions!

Cobb-Douglas: Constant Returns Case

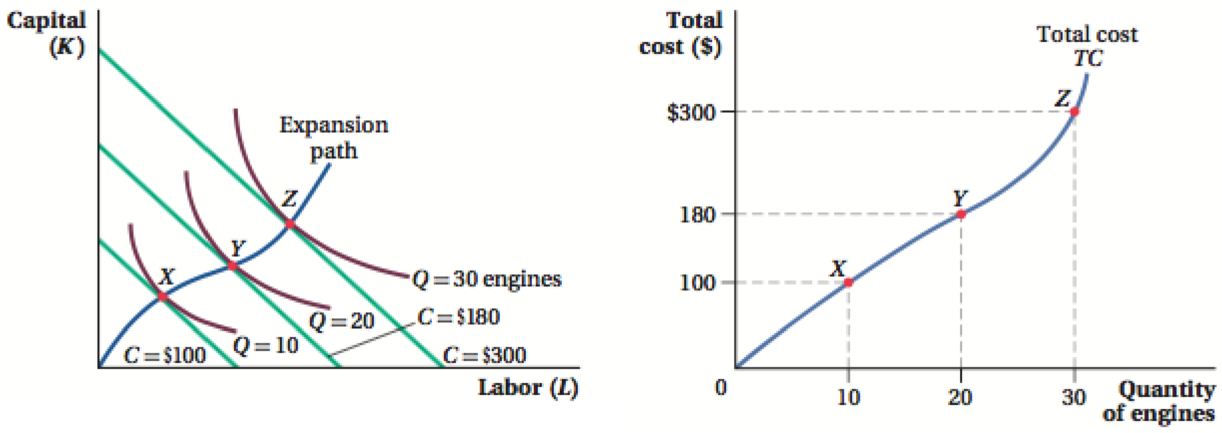
- A common case of Cobb-Douglas is often written as:

$$q = Ak^{\alpha}l^{1-\alpha}$$

(i.e., the exponents sum to 1, constant returns)

- α is the **output elasticity of capital**
    - A 1% increase in k leads to an α% increase in q
- $1-\alpha$ is the **output elasticity of labor**
    - A 1% increase in l leads to a $(1-\alpha)$% increase in q

Output-Expansion Paths & Cost Curves



Goolsbee et. al (2011: 246)

- **Output Expansion Path**: curve illustrating the changes in the optimal mix of inputs and the total cost to produce an increasing amount of output
- **Total Cost curve**: curve showing the total cost of producing different amounts of output (next class)
- See next class' notes page to see how we go from our least-cost combinations over a range of outputs to derive a total cost function